CHAPTER 4

Unsymmetrical Fault Analysis

4.1 Introduction

Practical systems rarely have perfectly balanced loads, currents, voltages or impedances in all the three phases. The analysis of unbalanced cases is greatly simplified by the use of the techniques of symmetrical components. The method of symmetrical components was developed by C.L. Fortescue prior to 1920.

4.2 Symmetrical Components

Symmetrical components or Fortescue theorem: It has been proven that an unbalanced system of n related phasors can be resolved into n systems of balanced phasors referred to as symmetrical components of the original phasors.

4.2.1 Sequence Operator '*a*'

The phase sequence of the phasors or vectors is the order in which they pass through a positive maximum. Thus phase sequence *abc* implies that the maximum occur in the order a, b, c or R, Y, B.

When the balanced components are considered, we see that the most frequently occurring angle is 120°.

In complex number theory, we defined j as the complex operator which is equal to $\sqrt{-1}$ and a magnitude of unity, and more importantly, when operated on any complex number rotates it anticlockwise by an angle of 90°, i.e. $j = \sqrt{-1} = 1\angle 90°$.

Similarly, we define a new complex operator a which has a magnitude of unity and when operated on any complex number rotates it anticlockwise by an angle of 120°.

i.e. $a = 1\angle 120° = -0.5 + j\ 0.866$

Some properties of a

$a = 1\angle 120°$
$a^2 = 1\angle 240°$ or $1\angle -120° = -0.5 - j0.866$
$a^3 = 1\angle 360°$ or 1
$1 + a + a^2 = 0$

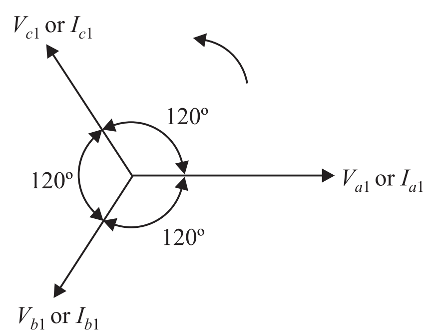
4.2.2 Types of Symmetrical Components

The unbalanced three-phase systems can be split up into the three balanced components, namely

- Positive sequence components
- Negative sequence components
- Zero sequence components

Positive sequence components: It consists of three phasors which are equal in magnitude, equally displaced 120° from each other and having the same phase sequence *abc*.

Let V_{a1}, V_{b1} and V_{c1} be the positive sequence voltages and I_{a1}, I_{b1} and I_{c1} be the positive sequence currents. It is assumed that the subscript 1 refers to the positive sequence.



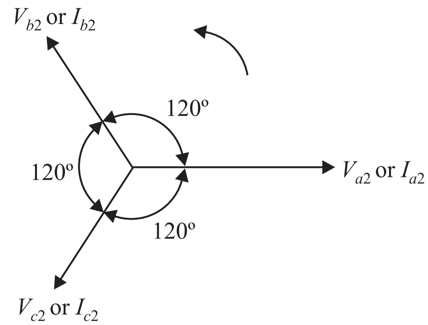
(a) Positive sequence components

$V_{a1} = V_{a1}\angle 0°$
$V_{b1} = V_{a1}\angle 240°$ or $V_{a1}\angle -120°$
$V_{c1} = V_{a1}\angle 120°$

$I_{a1} = I_{a1}\angle 0°$
$I_{b1} = I_{a1}\angle 240°$ or $I_{a1}\angle -120°$
$I_{c1} = I_{a1}\angle 120°$

Negative sequence components: It consists of three phasors which are equal in magnitude, equally displaced 120° from each other and having the same phase sequence as opposite to *acb*.

Let V_{a2}, V_{b2} and V_{c2} be the negative sequence voltages and I_{a2}, I_{b2} and I_{c2} be the negative sequence currents. It is assumed that the subscript 2 refers to the negative sequence.

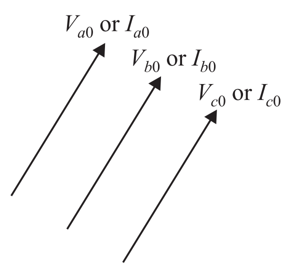


(b) Negative sequence components

$$V_{a2} = V_{a2}\angle 0° \qquad I_{a2} = I_{a2}\angle 0°$$
$$V_{b2} = V_{a2}\angle 120° \qquad I_{b2} = I_{a2}\angle 120°$$
$$V_{c2} = V_{a2}\angle 240° \text{ or } V_{a2}\angle -120° \qquad I_{c2} = I_{a2}\angle 240° \text{ or } I_{a2}\angle -120°$$

Zero sequence components: It consists of three phasors which are equal in magnitude, and zero phase displacement from each other.

Let V_{a0}, V_{b0} and V_{c0} be the zero sequence voltages and I_{a0}, I_{b0} and I_{c0} be the zero sequence currents. It is assumed that the subscript 0 refers to the zero sequence.



(c) Zero sequence components

Figure 4.1 Phasor diagram of symmetrical components.

$$V_{a0} = V_{b0} = V_{c0} \quad \text{or} \quad I_{a0} = I_{b0} = I_{c0}$$

4.2.3 Determination of Unbalanced Vectors from Their Symmetrical Components

Let V_a, V_b and V_c represent an unbalanced set of voltage phasors. Figure 4.1 (a, b and c) shows three such set of symmetrical components. Since each of the original unbalanced phasors is the sum of its components, the original phasors expressed in terms of their components are

$$V_a = V_{a0} + V_{a1} + V_{a2} \tag{4.1}$$
$$V_b = V_{b0} + V_{b1} + V_{b2} \tag{4.2}$$
$$V_c = V_{c0} + V_{c1} + V_{c2} \tag{4.3}$$

From the phasor diagram shown in the Figure 4.1 (a, b and c), we get

$$V_{a0} = V_{b0} = V_{c0} \tag{4.4}$$

$$V_{b1} = V_{a1}\angle 240° = a^2 V_{a1} \qquad V_{b2} = V_{a1}\angle 120° = a^2 V_{a2} \tag{4.5}$$

$$V_{c1} = V_{a1}\angle 120° = aV_{a1} \qquad V_{c2} = V_{a1}\angle 240° = a^2 V_{a2} \tag{4.6}$$

Repeating Eq. (4.1) and substituting Eqs. (4.4), (4.5) and (4.6) in Eqs. (4.2) and (4.3),

$$V_a = V_{a0} + V_{a1} + V_{a2} \tag{4.7}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2} \tag{4.8}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2} \tag{4.9}$$

In the matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{4.10}$$

The above equation can be used to calculate the unbalanced voltage vectors from their symmetrical components.

4.2.4 Determination of Symmetrical Components of Unbalanced Vectors

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [A] \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{4.11}$$

where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Premultiplying Eq. (4.11) by A^{-1} yields

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = [A]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{4.12}$$

As these relations are important, therefore we can write the separate equations in the expanded form

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c] \tag{4.13}$$

$$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c] \tag{4.14}$$

$$V_{a2} = \frac{1}{3}[V_a + a^2V_b + aV_c] \tag{4.15}$$

The above equations can be used to calculate the symmetrical components of the unbalanced voltages.

The preceding equations could have been written for any set of related phasors, and we might have them currents instead of voltages. They are summarized for currents as follows.

$$I_a = I_{a0} + I_{a1} + I_{a2} \tag{4.16}$$

$$I_b = I_{a0} + a^2I_{a1} + aI_{a2} \tag{4.17}$$

$$I_c = I_{a0} + aI_{a1} + a^2I_{a2} \tag{4.18}$$

In the matrix form

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.19}$$

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c] \tag{4.20}$$

$$I_{a1} = \frac{1}{3}[I_a + aI_b + a^2I_c] \tag{4.21}$$

$$I_{a2} = \frac{1}{3}[I_a + a^2I_b + aI_c] \tag{4.22}$$

In the matrix form

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.23}$$

EXAMPLE 4.1 In a 3-phase 4-wire system, the currents in *R*, *Y* and *B* lines under abnormal conditions of loading are as under:

$$I_R = 100\angle 30° \text{ A}; \quad I_Y = 50\angle 300° \text{ A}; \quad I_B = 30\angle 180° \text{ A}$$

Calculate the positive, negative and zero sequence currents in the *R* line and return current in the neutral wire.

Solution: Let I_{R0}, I_{R1} and I_{R2} be the zero, positive and negative sequence currents respectively of the line current in red line.

$$I_{R0} = \frac{1}{3}[I_R + I_Y + I_B]$$
$$= \frac{1}{3}[100\angle 30° + 50\angle 300° + 30\angle 180°]$$
$$= \frac{1}{3}[(86.60 + j50) + (25 - j43.3) + (-30 + j0)]$$
$$= \frac{1}{3}(81.60 + j6.7) = 27.2 + j2.23 = 27.29\angle 4.68°\text{A}$$

$$I_{R1} = \frac{1}{3}[I_R + aI_Y + a^2 I_B]$$
$$= \frac{1}{3}[100\angle 30° + 1\angle 120° \times 50\angle 300° + 1\angle -120° \times 30\angle 180°]$$
$$= \frac{1}{3}[100\angle 30° + 50\angle 420° + 30\angle 60°]$$
$$= \frac{1}{3}[(86.60 + j50) + (25 + j43.3) + (15 + j25.98)]$$
$$= \frac{1}{3}(126.6 + j119.28) = 42.2 + j39.76 = 57.98\angle 43.3°\text{A}$$

$$I_{R2} = \frac{1}{3}[I_R + a^2 I_Y + aI_B]$$
$$= \frac{1}{3}[100\angle 30° + 1\angle -120° \times 50\angle 300° + 1\angle 120° \times 30\angle 180°]$$
$$= \frac{1}{3}[100\angle 30° + 50\angle 180° + 30\angle 300°]$$
$$= \frac{1}{3}[(86.60 + j50) + (-50 + j0) + (15 - j25.98)]$$
$$= \frac{1}{3}(51.6 + j24.02) = 17.2 + j8.007 = 18.97\angle 24.96°\text{A}$$

EXAMPLE 4.2 The symmetrical components of a set of unbalanced three-phase currents are:

$$I_{a0} = 100 \text{ A}; \quad I_{a1} = 200 - j100 \text{ A}; \quad I_{a2} = -100 \text{ A}$$

Calculate the original unbalanced phasors.

Solution: To find the unbalanced vectors

$$I_a = I_{a0} + I_{a1} + I_{a2}$$
$$= [100 + 200 - j100 - 100]$$
$$= 200 - j100 = 223.6\angle -26.56° \text{ A}$$
$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$$

$= [100\angle 0° + 1\angle{-120°} \times 223.6\angle{-26.56°} + 1\angle 120° \times 100\angle 180°]$

$= [100\angle 0° + 223.65\angle 213.44° + 100\angle 300°]$

$= -36.58 + j209.8 = 213\angle 99.89°$ A

$I_c = I_{a0} + aI_{a1} + a^2 I_{a2}$

$= [100\angle 0° + 1\angle 120° \times 223.6\angle{-26.56°} + 1\angle{-120°} \times 100\angle 180°]$

$= [100\angle 0° + 223.65\angle 93.44° + 100\angle 420°]$

$= 136.6 + j309.8 = 338.57\angle 66.2°$ A

EXAMPLE 4.3 A delta connected balance resistive load is connected across an unbalanced three-phase supply shown in Figure 4.2 with currents in line *a* and *b* specified. Determine the symmetrical components of the currents.

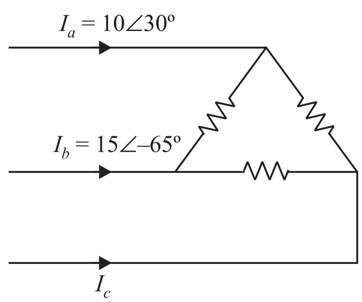


Figure 4.2

Solution:

$$I_a = 10\angle 30°$$
$$I_b = 15\angle{-65°}$$

In a balanced load

$$I_a + I_b + I_c = 0$$

$$I_c = -I_a - I_b$$

$$I_c = -(10\angle 30°) - (15\angle{-65°}) = 17.3\angle 150° \text{A}$$

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$$

$$= \frac{1}{3}[10\angle 30° + 15\angle{-65°} + 17.3\angle 150°]$$

$$= \frac{1}{3}(0) = 0 \text{ A}$$

$$I_{a1} = \frac{1}{3}[I_a + aI_b + a^2 I_c]$$

$$= \frac{1}{3}[10\angle 30° + 1\angle 120° \times 15\angle{-65°} + 1\angle{-120°} \times 17.3\angle 150°]$$

$$= \frac{1}{3}(33.38\angle 15°) = 11.13\angle 15° \text{ A}$$

$$I_{a2} = \frac{1}{3}[I_a + a^2 I_b + aI_c]$$
$$= \frac{1}{3}[10\angle 30° + 1\angle -120° \times 15\angle -65° + 1\angle 120° \times 17.3\angle 150°]$$
$$= \frac{1}{3}(12.66\angle -119.7°) = 4.22\angle -119.7° \text{A}$$

4.2.5 Power in Symmetrical Components

The symmetrical components of voltages and currents are known, the power in a three-phase circuit can be determined from the symmetrical components. The total complex power flowing into three-phase circuit in all the three-phase lines a, b, c is

$$S = P + jQ = VI^* = V_a I_a^* + V_b I_b^* + V_c I_c^* \tag{4.24}$$

where V_a, V_b, V_c are the phase voltages and I_a, I_b, I_c are the phase currents. The above equation can be written in the matrix form as

$$S = [V_a \quad V_b \quad V_c]\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* \tag{4.25}$$

Substituting Eqs. (4.10) and (4.19) in the following equation

$$S = [AV]^T [AI]^* \tag{4.26}$$

where

$$V = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}; \qquad I = \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$S = A^T V^T A^* I^* = V^T A^T A^* I^* \qquad [\therefore A^T = A]$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$A \cdot A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3[U]$$

$$= 3 \text{ [3 × 3 Unit matrix]}$$

The power

$$S = V^T A^T A^* \mathrm{I}^* = V^T 3[U] I^*$$

$$S = 3[V]^T [I]^* = 3[V_{a0} \quad V_{a1} \quad V_{a2}] \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$S = 3[V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^*] \tag{4.27}$$

The total unbalanced power can be obtained from the sum of symmetrical components of power.

4.3 Sequence Impedance

Sequence impedance: The sequence impedances are the impedances offered by the power system components or elements to positive, negative and zero sequence currents.

Positive sequence impedance: The impedance of a circuit element when positive sequence currents alone are flowing is called the positive sequence impedance.

Negative sequence impedance: The impedance of a circuit element when negative sequence currents alone are flowing is called the negative sequence impedance.

Zero sequence impedance: The impedance of a circuit element when zero sequence currents alone are flowing is called the zero sequence impedance.

4.4 Sequence Network of Power System Components

The single-phase equivalent circuit of power system consisting of impedances to current of any one sequence only is called sequence network.

4.4.1 Sequence Network of Unloaded Generator

Let us consider three-phase circuit diagram of unloaded generator as shown in Figure 4.3. The neutral of the generator is grounded through impedance.

The positive sequence reactance of a generator may be X_d or X_d' or X_d'' depending upon the condition at which the reactance is calculated with positive sequence voltages applied. When negative sequence currents are impressed on the stator winding, the net flux rotates at twice the synchronous speed relative to the rotor. The negative sequence reactance is approximately given by $X_2 = X_d''$. The zero sequence currents, when they flow, are identical and the spatial distribution of the mmfs is sinusoidal. The resultant air gap flux due to zero sequence currents is zero. Thus, the zero sequence reactance is approximately the same as the leakage flux $X_0 = X$.

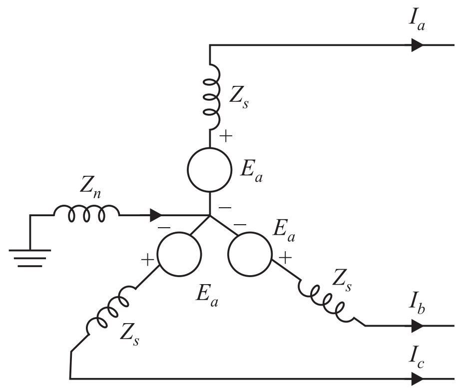


Figure 4.3 Circuit diagram of unloaded generator grounded through impedance.

4.4.2 Sequence Network of Loaded Generator

Figure 4.4 represents a three phase synchronous generator with neutral is grounded through an impedance Z_n. The synchronous generator is supplying a three phase balanced load.

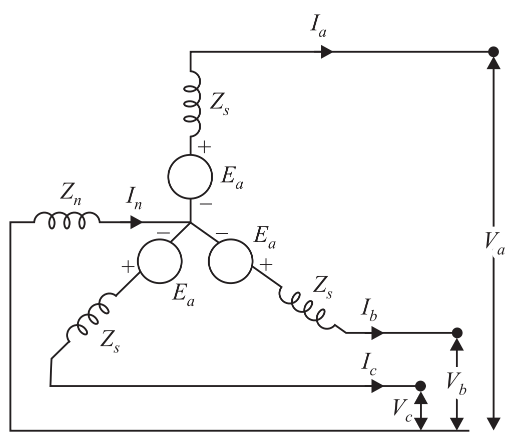


Figure 4.4 Circuit diagram of loaded synchronous generator neutral is grounded through impedance.

$$V_a = E_a - I_a Z_s - I_n Z_n \tag{4.28}$$
$$V_b = E_a - I_b Z_s - I_n Z_n \tag{4.29}$$
$$V_c = E_a - I_c Z_s - I_n Z_n \tag{4.30}$$
$$I_n = I_a + I_b + I_c \tag{4.31}$$

Substituting Eq. (4.31) in Eq. (4.28)

$$V_a = E_a - I_aZ_s - (I_a + I_b + I_c)\,Z_n$$
$$V_a = E_a - I_aZ_s - I_aZ_n - I_bZ_n - I_cZ_n$$
$$V_a = E_a - I_a(Z_s + Z_n) - (I_b + I_c)\,Z_n \tag{4.32}$$

Similarly

$$V_b = E_a - I_b(Z_s + Z_n) - (I_a + I_c)\,Z_n \tag{4.33}$$
$$V_c = E_a - I_c(Z_s + Z_n) - (I_b + I_a)\,Z_n \tag{4.34}$$

In the matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_a \\ E_a \end{bmatrix} - \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.35}$$

$$V^{abc} = E^{abc} - Z^{abc} I^{abc} \tag{4.36}$$

where V^{abc} is the phase terminal voltage vector and I^{abc} is the phase current vector. Converting the terminal voltages and current phasors into their symmetrical components result in

$$[A]\,[V^{012}] = [A]\,[E^{012}] - [A]\,[Z^{abc}]\,[I^{012}]$$
$$[A]^{-1}\,[A]\,[V^{012}] = [A]^{-1}\,[A]\,[E^{012}] - [A]^{-1}\,[A]\,[Z^{abc}]\;[I^{012}]$$
$$V^{012} = E^{012} - Z^{012} I^{012} \tag{4.37}$$

where

$$Z^{012} = A^{-1} Z^{abc} A$$

$$Z^{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \times \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Performing the above multiplications, we get

$$Z^{012} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \tag{4.38}$$

Since the generated e.m.f is balanced, we have to take only the positive sequence voltage E_a

$$E^{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} \tag{4.39}$$

Substituting for Z^{012} and E^{012} in Eq. (4.37)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.40}$$

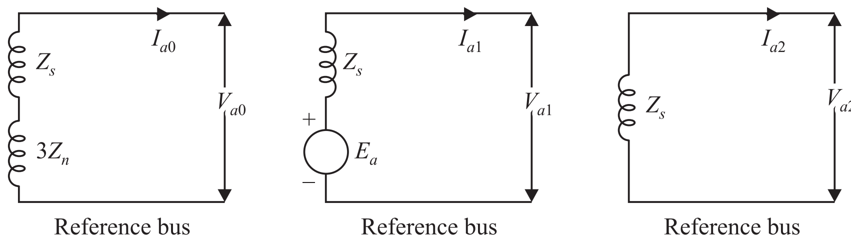
From the above equation

$$V_{a0} = -(Z_s + 3Z_n)I_{a0} \tag{4.41}$$

$$V_{a1} = E_a - (Z_s)I_{a1} \tag{4.42}$$

$$V_{a2} = -(Z_s)I_{a2} \tag{4.43}$$

These may be expressed in the network form as shown in Figures 4.5.



(a) Zero sequence network (b) Positive sequence network (c) Negative sequence network

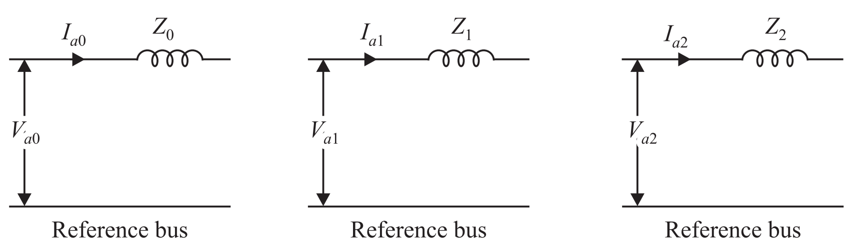
Figure 4.5 Sequence network of a loaded generator.

4.4.3 Sequence Network of Transmission Line

The conductors of a transmission line, being passive and stationary, do not have an inherent direction. The transmission line (or cable) may be represented by a single reactance in the single-line diagram.

Thus they always have the same positive sequence impedance and negative sequence impedance. However, as the zero sequence paths also involve the earth wire and the earth return path, the zero sequence impedance is higher in value.

The zero, positive and zero sequence impedances of transmission lines are represented as a series impedance in their respective sequence networks as shown in Figure 4.6.



(a) Zero sequence network (b) Positive sequence network (c) Negative sequence network

Figure 4.6 Sequence network of a transmission line.

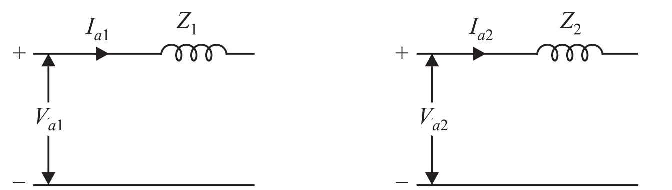
Typically, the ratio of the zero sequence impedance to the positive sequence impedance would be of the order of 2 for a single circuit transmission line with earth wire, about 3.5 for a single circuit with no earth wire or for a double circuit line.

For a single core cable, the ratio of the zero sequence impedance to the positive sequence impedance would be around 1 to 1.25.

Transmission lines are assumed to be symmetrical in all three phases. However, this assumption would not be valid for long untransposed lines (say, beyond 500 km) as the mutual coupling between the phases would be unequal, and then symmetrical components cannot be used.

4.4.4 Sequence Network of Transformer

The transformer too, being passive and stationary, does not have an inherent direction. Thus it always has the same positive sequence impedance, negative sequence impedance and even the zero sequence impedance. However, the zero sequence paths across the windings of a transformer depend on the winding connections and even grounding impedance. The positive and negative sequence networks of a transformer are shown in Figure 4.7.

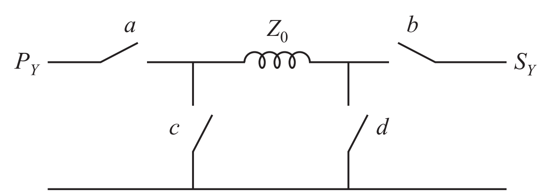


(a) Positive sequence network (b) Negative sequence network

Figure 4.7 Positive and negative sequence networks of a transformer.

Zero sequence network of three phase transformers

However, the zero sequence paths across the windings of transformer depend on the different winding connections and even grounding impedance. The zero sequence network of three-phase transformer can be easily constructed by considering the arrangement as follows.



S.No.	*Primary winding (P_Y)*	*Secondary winding (S_Y)*
1.	Grounded star—close '*a*'	Grounded star—close '*b*'
2.	Delta—close '*c*'	Delta—close '*d*'
3.	Ungrounded—open '*a*', '*c*'	Ungrounded—open '*b*', '*d*'

The zero sequence network of three-phase transformers is shown in Figure 4.8.

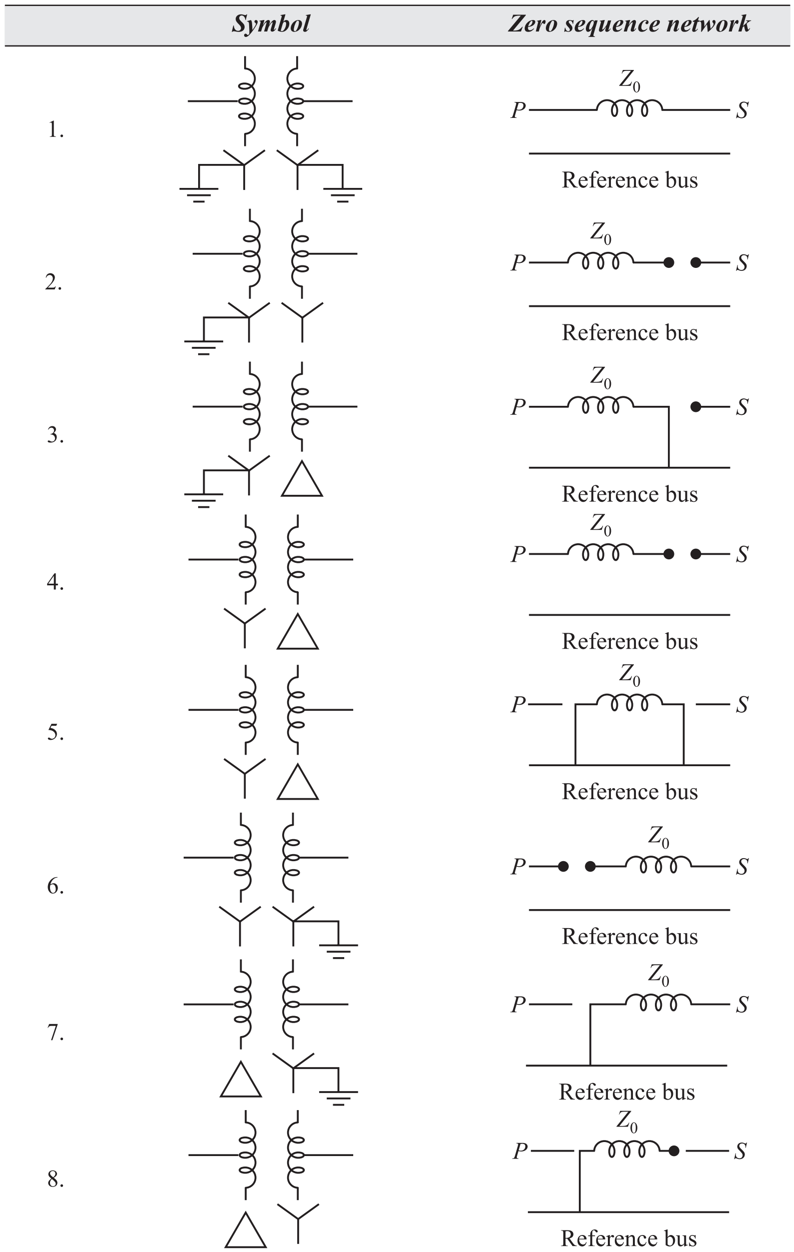


Figure 4.8 Zero sequence network of three-phase transformer.

4.4.5 Sequence Network of Star Connected Load Grounded Through Impedance

A three-phase balanced load with self and mutual elements is shown in Figure 4.9. The load neutral is grounded through an impedance Z_n.

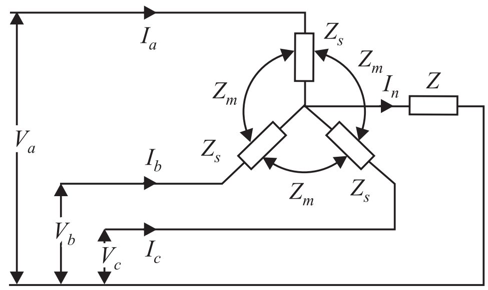


Figure 4.9 Circuit diagram of star connected load grounded through Z_n.

Lines to neutral voltages are

$$V_a = I_a Z_s + I_b Z_m + I_c Z_m + I_n Z_n \tag{4.44}$$

$$V_b = I_a Z_m + I_b Z_s + I_c Z_m + I_n Z_n \tag{4.45}$$

$$V_c = I_a Z_m + I_b Z_m + I_c Z_s + I_n Z_n \tag{4.46}$$

From KCL, we have

$$I_n = I_a + I_b + I_c \tag{4.47}$$

Substituting for I_n from Eq. (4.47) into Eqs. (4.44) to (4.46) and rewriting these equations in matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_s & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.48}$$

$$V^{abc} = Z^{abc} I^{abc} \tag{4.49}$$

where V^{abc} is the phase terminal voltage vector and I^{abc} is the phase current vector. Converting the terminal voltages and current phasors into their symmetrical components result in

$$[A][V^{012}] = [A][Z^{abc}][I^{012}]$$

$$[A]^{-1}[A][V^{012}] = [A]^{-1}[A][Z^{abc}][I^{012}]$$

$$V^{012} = Z^{012} I^{012} \tag{4.50}$$

where

$$Z^{012} = A^{-1} Z^{abc} A$$

$$Z^{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Performing the above multiplications, we get

$$Z^{012} = \begin{bmatrix} Z_s + 3Z_n + 2Z_m & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \tag{4.51}$$

When there is no mutual coupling, $Z_m = 0$. Therefore,

$$Z^{012} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \tag{4.52}$$

Substituting for Z^{012} in Eq. (4.50)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.53}$$

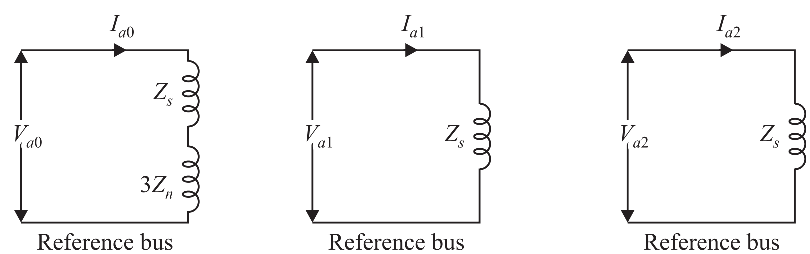
From the above equation

$$V_{a0} = (Z_s + 3Z_n)I_{a0} \tag{4.54}$$

$$V_{a1} = Z_s I_{a1} \tag{4.55}$$

$$V_{a2} = Z_s I_{a2} \tag{4.56}$$

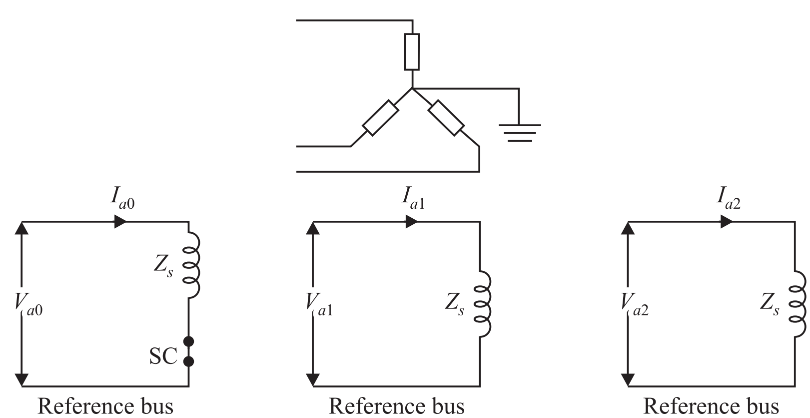
These may be expressed in network form as shown in Figure 4.10.



(a) Zero sequence network (b) Positive sequence network (c) Negative sequence network

Figure 4.10 Sequence network of a star connected load grounded through impedance.

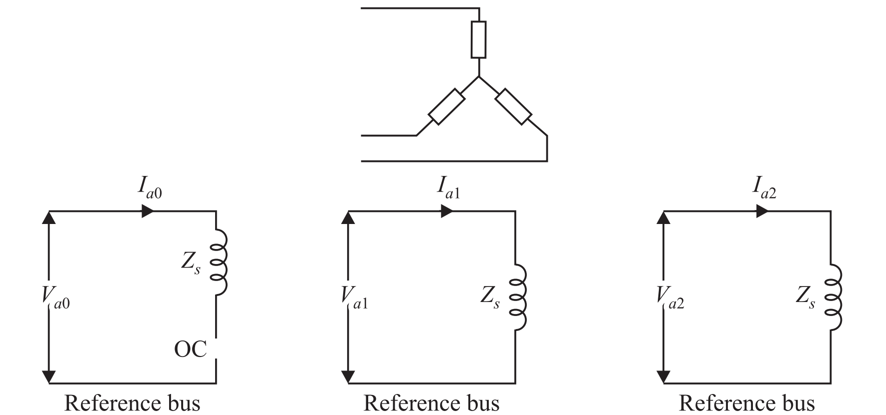
Sequence network of three-phase balance star connected load with solid grounded



(a) Zero sequence network (b) Positive sequence network (c) Negative sequence network

Figure 4.11 Sequence network of a star connected load with solid grounded.

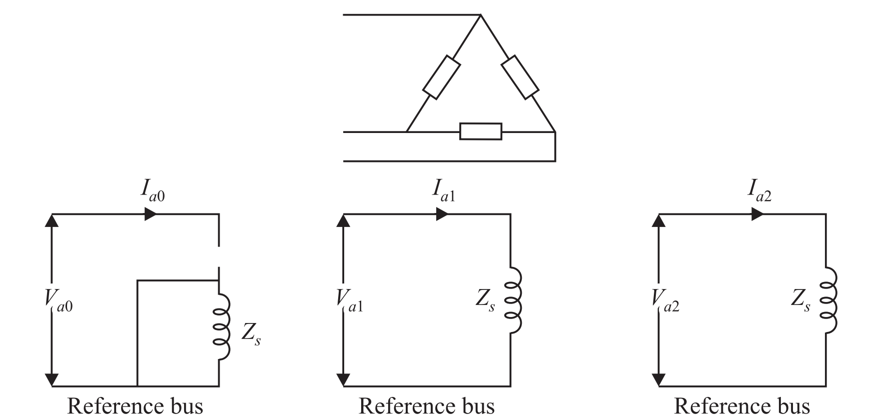
Sequence network of three-phase balance star connected load ungrounded



(a) Zero sequence network (b) Positive sequence network (c) Negative sequence network

Figure 4.12 Sequence network of a star connected load ungrounded.

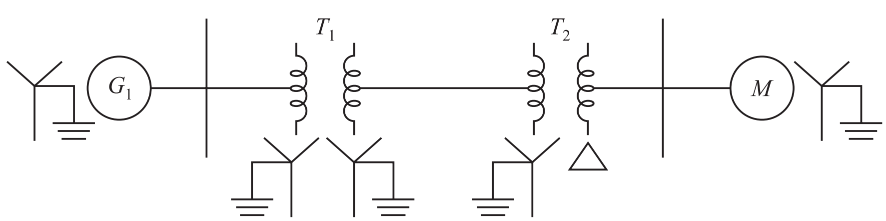
Sequence network of three-phase balance delta connected load



(a) Zero sequence network (b) Positive sequence network (c) Negative sequence network

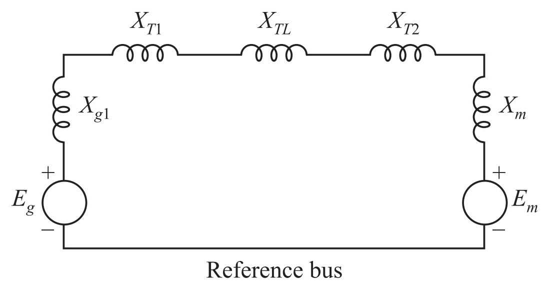
Figure 4.13 Sequence network of a delta connected load.

EXAMPLE 4.4 Draw the positive, negative and zero sequence impedance diagram.

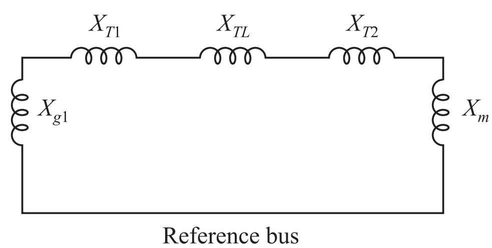


Solution:

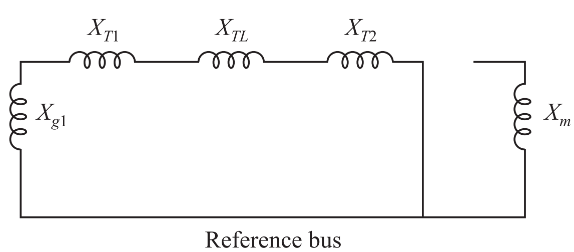
Positive sequence impedance diagram



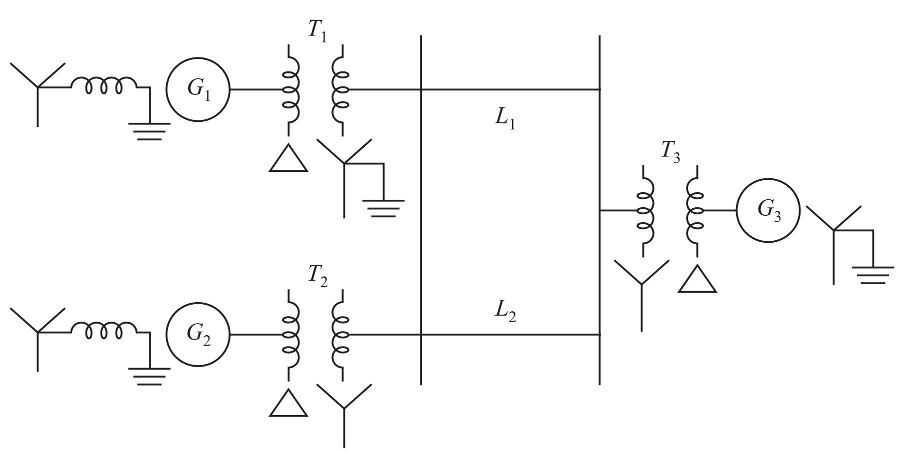
Negative sequence impedance diagram



Zero sequence impedance diagram

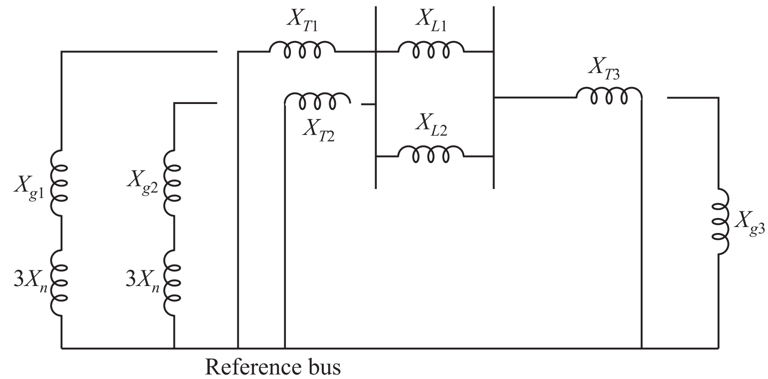


EXAMPLE 4.5 Draw the zero sequence networks for the system.

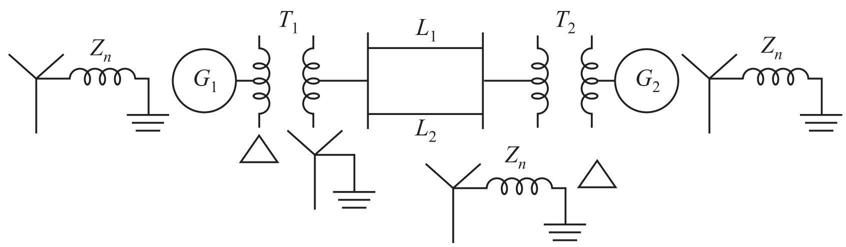


Solution:

Zero sequence impedance diagram

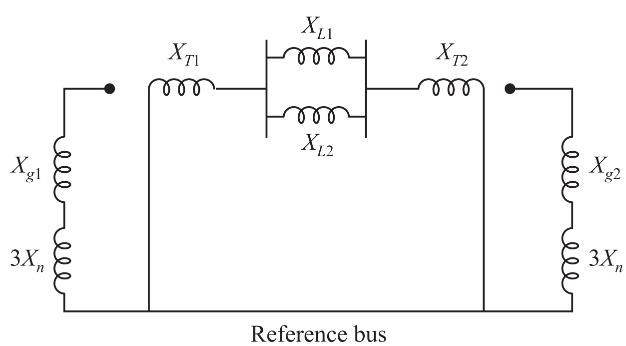


EXAMPLE 4.6 Draw the zero sequence networks for the system.



Solution:

Zero sequence impedance diagram



4.5 Unsymmetrical Faults

On the occurrence of a fault, current and voltage conditions become abnormal, the delivery of power to the loads may be unsatisfactory over a considerable area, and if the faulted equipment is not promptly disconnected from the remainder of the system, damage may result to other pieces of operating equipment. Most of the faults that occur on power system are single line to ground faults, line to line faults and double line to ground faults, with and without fault impedance.

While the unbalanced currents are caused by unsymmetrical faults, the method of symmetrical components is used to determine the currents and voltages in all parts of the power system after the occurrence of the fault.

In this topic we shall first discuss faults at the terminals of unloaded synchronous generator with fault impedance. Then we shall consider faults on power system by applying Thevenin's theorem, which allows us to find the fault current by replacing the entire system by a single generator and series impedance.

Important assumptions of power system representation

(i) Power system operates under balanced steady state conditions before the fault occurs. Therefore, the positive, negative and zero sequence networks are uncoupled before the occurrence of the fault. When an unsymmetrical fault occurs, they get interconnected at the point of fault.
(ii) Prefault load current at the point of fault is generally neglected. Positive sequence voltages of all the three phases are equal to the prefault voltage.
(iii) Transformer winding resistances and shunt admittances are neglected.
(iv) Transmission line series resistances and shunt admittances are neglected.
(v) Synchronous machine armature resistance, saliency and saturation are neglected.
(vi) Induction motors are either neglected or represented as synchronous machines.

Types of unsymmetrical faults

1. Single line to ground fault (L–G fault)
2. Line to line fault (L–L fault)
3. Double line to ground fault (L–L–G fault)

4.5.1 Single Line to Ground Fault (L–G Fault)

Let us consider three-phase circuit diagram of unloaded generator shown in Figure 4.14. The neutral of the generator is grounded through impedance.

Suppose a single line to ground fault occurs on phase a through impedance Z_f. Assuming the generator is initially on no load, the boundary conditions at the fault point are

(i) $V_a = Z_f I_a$ (4.57)
(ii) $I_b = I_c = 0$ (4.58)
(iii) Fault current $I_f = I_a$ (4.59)

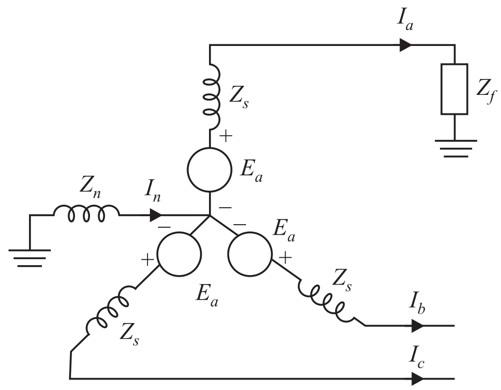


Figure 4.14 Single line to ground fault through Z_f on phase a of unloaded generator.

The symmetrical components of currents from Eq. (4.23) can be rewritten as

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.60}$$

Substituting Eq. (4.58) in Eq. (4.60), we get

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{4.61}$$

From the above equation, we find that

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \tag{4.62}$$

Rewriting Eq. (4.40)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.63}$$

where zero sequence impedance, $Z_0 = Z_s + 3Z_n$
positive sequence impedance, $Z_1 = Z_s$
negative sequence impedance, $Z_2 = Z_s$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.64}$$

From the above equation

$$V_{a0} = -Z_0 I_{a0} \tag{4.65}$$

$$V_{a1} = E_a - Z_1 I_{a1} \tag{4.66}$$

$$V_{a2} = -Z_2 I_{a2} \tag{4.67}$$

Rewriting Eq. (4.10)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{4.68}$$

From the above equation

$$V_a = V_{a0} + V_{a1} + V_{a2} \tag{4.69}$$

Substituting Eqs. (4.65), (4.66) and (4.67) in Eq. (4.69), we get

$$V_a = -Z_0 I_{a0} + (E_a - Z_1 I_{a1}) - Z_2 I_{a2} \tag{4.70}$$

$$V_a = E_a - (Z_0 I_{a0} + Z_1 I_{a1} + Z_2 I_{a2}) \tag{4.71}$$

Substituting Eq. (4.62) in Eq. (4.71), we get

$$V_a = E_a - \left(Z_0 \frac{I_a}{3} + Z_1 \frac{I_a}{3} + Z_2 \frac{I_a}{3} \right) \tag{4.72}$$

$$V_a = E_a - \frac{I_a}{3}(Z_0 + Z_1 + Z_2) \tag{4.73}$$

From the boundary conditions (i) $V_a = Z_f I_a$

$$E_a - \frac{I_a}{3}(Z_0 + Z_1 + Z_2) = Z_f I_a$$

$$3E_a - I_a(Z_0 + Z_1 + Z_2) = 3Z_f I_a$$

Therefore the fault current

$$I_f = I_a = \frac{3E_a}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{4.74}$$

or

From Eq. (4.62)

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_a}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{4.75}$$

Suppose the fault impedance $Z_f = 0$ (direct short-circuit)

$$I_f = I_a = \frac{3E_a}{Z_0 + Z_1 + Z_2} \tag{4.76}$$

Equations (4.62) and (4.76) can be represented by connecting the sequence network in series as shown in the equivalent circuit of Figure 4.15. Thus, for line to ground faults, the Thevenin's impedance to the point of fault is obtained from each sequence network, and the three sequence networks are placed in series. In many practical applications, Z_1 and Z_2 are the same. If the neutral of the synchronous generator is solidly grounded, $Z_n = 0$.

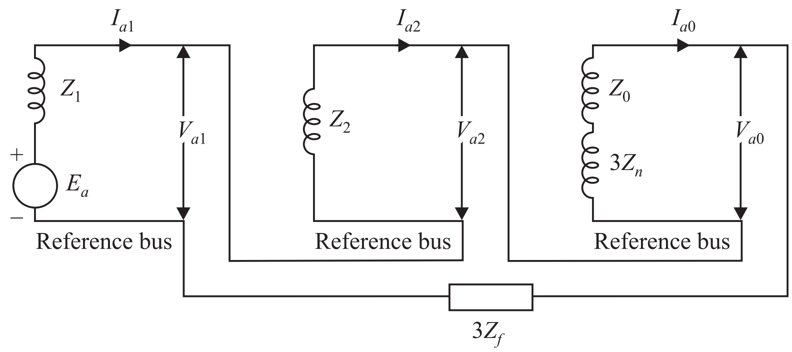


Figure 4.15 Sequence network of single line to ground fault.

EXAMPLE 4.7 A 30 MVA, 11 kV, 3Φ synchronous generator has a direct subtransient reactance of 0.25 p.u. The negative and zero sequence reactance are 0.35 and 0.1 p.u. respectively. The neutral of the generator is solidly grounded. Determine the subtransient current in the generator and the line to line voltages for subtransient conditions when a single line to ground fault occurs at the generator terminals with the generator operating unloaded at rated voltage.

Solution: $E_a = 1$ p.u.

Direct subtransient reactance, $X_d'' = Z_1 = j0.25$ p.u.

$$X_2 = Z_2 = j0.35 \text{ p.u.}$$
$$X_0 = Z_0 = j0.1 \text{ p.u.}$$
$$Z_f = 0$$

$$\text{Fault current } I_f = I_a = \frac{3E_a}{Z_0 + Z_1 + Z_2 + 3Z_f}$$

$$= \frac{3 \times 1.0}{j0.1 + j0.25 + j0.35} = -\,j4.2857 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\,\text{kV}_b} \times 10^3 = \frac{30}{\sqrt{3} \times 11} \times 10^3 = 1574.59 \text{ A}$$

Fault current in A = Fault current in p.u. (I_f) × base current

Fault current A, $|I_f| = 4.2857 \times 1574.59 = 6748.22$ A

The symmetrical components of the voltages from point *a* to ground are:

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_a}{Z_0 + Z_1 + Z_2 + 3Z_f} = \frac{1.0}{j0.1 + j0.25 + j0.35} = -\,j1.4286 \text{ p.u.}$$

$$V_{a0} = -Z_0 I_{a0} = -(j0.1)\,(-j1.4286) = -0.143 \text{ p.u.}$$

$$V_{a1} = E_a - Z_1 I_{a1} = 1.0 - (j0.25)\,(-j1.4286) = 0.643 \text{ p.u.}$$

$$V_{a2} = -Z_2 I_{a2} = -(j0.35)\,(-j1.4286) = -0.50 \text{ p.u.}$$

Line to ground voltages are

$$V_a = V_{a0} + V_{a1} + V_{a2} = -0.143 + 0.643 - 0.50 = 0$$

$$V_b = V_{a0} + a^2V_{a1} + aV_{a2}$$
$$= -0.143 + 1\angle{-120°} \times 0.643 + 1\angle 120° \times (-0.50)$$
$$= -0.143 + (-0.5 - j0.866) \times 0.643 + (-0.5 + j0.866) \times (-0.50)$$
$$= -0.215 - j0.989 \text{ p.u.}$$

$$V_c = V_{a0} - aV_{a1} + a^2V_{a2}$$
$$= -0.143 + 1\angle 120° \times 0.643 + 1\angle{-120°} \times (-0.50)$$
$$= -0.143 + (-0.5 + j0.8666) \times 0.643 + (-0.5 - j0.866) \times (-0.50)$$
$$= -0.215 + j0.989 \text{ p.u.}$$

Line to line voltages are

$$V_{ab} = V_a - V_b = 0 - (-0.215 - j0.989) = 0.215 + j0.989$$
$$= 1.012\angle 77.7° \text{ p.u.}$$

$$V_{bc} = V_b - V_c = (-0.215 - j0.989) - (-0.215 + j0.989)$$
$$= 0 - j1.978 = 1.978\angle 270° \text{ p.u.}$$

$$V_{ca} = V_c - V_a = (-0.215 + j0.989) - (0)$$
$$= -0.215 + j0.989 = -1.012\angle 102.3° \text{ p.u.}$$

The above line voltages are expressed in per unit of the base voltage to neutral. Therefore, the post fault line voltages expressed in kV are

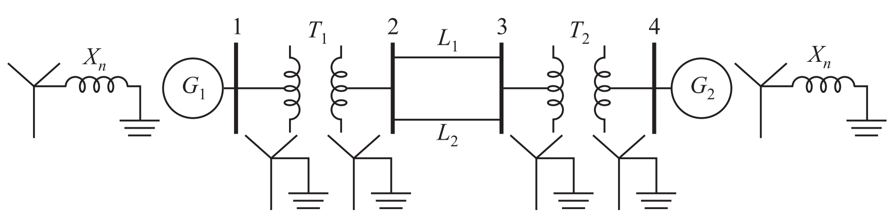
$$V_{ab} = 1.012\angle 77.7° \times \frac{11}{\sqrt{3}} = 6.427\ \angle 77.7° \text{ kV}$$

$$V_{bc} = 1.978\angle 270° \times \frac{11}{\sqrt{3}} = 12.562\ \angle 270° \text{ kV}$$

$$V_{ca} = -1.012\angle 102.3° \times \frac{11}{\sqrt{3}} = -6.427\ \angle 102.3° \text{ kV}$$

EXAMPLE 4.8 Determine the fault current and MVA at faulted bus for a line to ground fault at bus 4 as shown in the figure.

G_1 and G_2: 100 MVA, 11 kV, $X^+ = X^- = 15\%$; $X^0 = 5\%$ and $X_n = 6\%$
T_1 and T_2: 100 MVA, 11/220 kV, $X_{\text{leakage}} = 9\%$
L_1 and L_2: $X^+ = X^- = 10\%$; $X^0 = 10\%$ on a base 100 MVA. Consider a fault at a phase.



Solution:

Base MVA, MVA_{new} = 100 MVA

Base kV, kV_{new} = 220 kV

Positive and negative reactances of transmission lines L_1 and L_2

$X_{p.u.(given)} = 0.10$ p.u., $MVA_{given} = 100$, $MVA_{new} = 100$, $kV_{given} = 220$, $kV_{new} = 220$

$$X_{p.u.(new)} = X_{p.u.given} \times \left(\frac{kV_{given}}{kV_{new}}\right)^2 \times \left(\frac{MVA_{new}}{MVA_{given}}\right)$$

$$X_{p.u.(new)} = j0.1 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{100}{100}\right) = j0.1 \text{ p.}$$

Zero reactance of transmission lines L_1 and L_2

$X_{p.u.(given)} = 0.1$ p.u.

$$X_{p.u.(new)} = j0.1 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{100}{100}\right) = j0.1 \text{ p.u.}$$

Reactance of transformers T_1 and T_2 (secondary)

$X_{p.u.(given)} = 0.09$ p.u., $MVA_{given} = 100$, $MVA_{new} = 100$, $kV_{given} = 220$, $kV_{new} = 220$

$$X_{p.u.(new)} = j0.09 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{100}{100}\right) = j0.09 \text{ p.u.}$$

Base kV on LT side of transformer T_1

$$= \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$\text{Base kV on LT side of transformer } T_1 = 220 \times \frac{33}{220} = 33 \text{ kV}$$

$$kV_{new} = 33 \text{ kV}$$

Positive and negative reactances of generators G_1 and G_2

$X_{p.u.(given)} = 0.15$ p.u., $MVA_{given} = 100$, $MVA_{new} = 100$, $kV_{given} = 11$, $kV_{new} = ?$

Base kV on LT side of transformer T_1

$$= \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$\text{Base kV on LT side of transformer } T_1 = 220 \times \frac{11}{220} = 11 \text{ kV}$$

$$kV_{new} = 11 \text{ kV}$$

$$X_{p.u.(new)} = j0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{100}\right) = j0.15 \text{ p.u.}$$

Zero reactance of generators G_1 and G_2

$X_{\text{p.u.(given)}} = 0.05$ p.u.

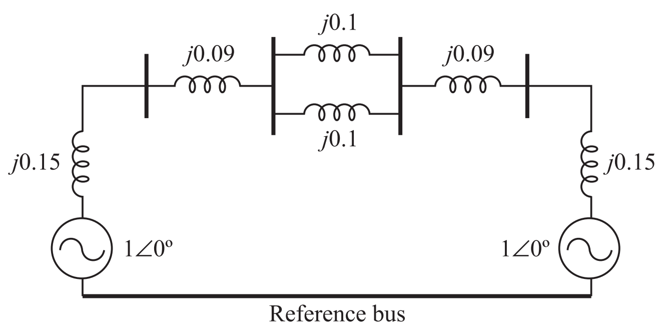
$$X_{\text{p.u.(new)}} = j0.05 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{100}\right) = j0.05 \text{ p.u.}$$

Neutral reactance of generators G_1 and G_2

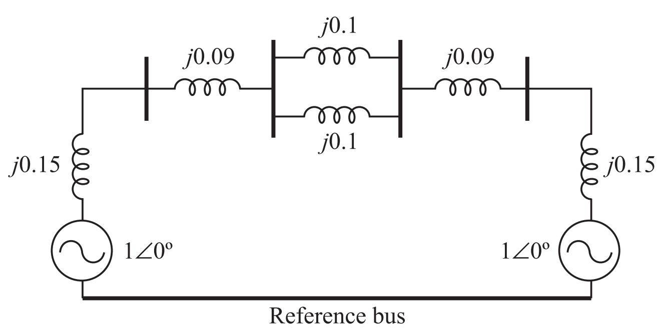
$X_{\text{p.u.(given)}} = 0.06$ p.u.

$$X_{\text{p.u.(new)}} = j0.06 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{100}\right) = j0.06 \text{ p.u.}$$

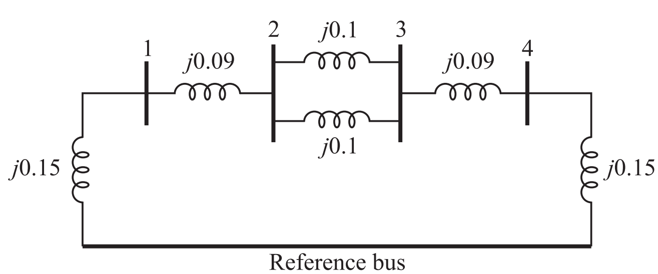
Reactance diagram

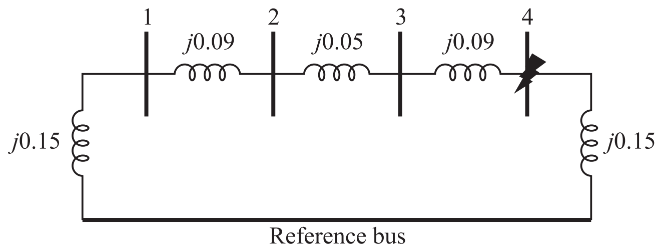


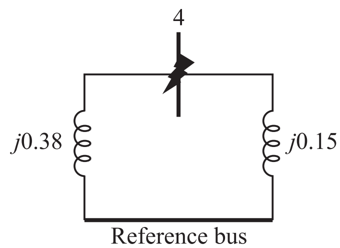
Positive sequence impedance diagram



Use Thevenin's theorem to find the positive sequence impedance Z_1.

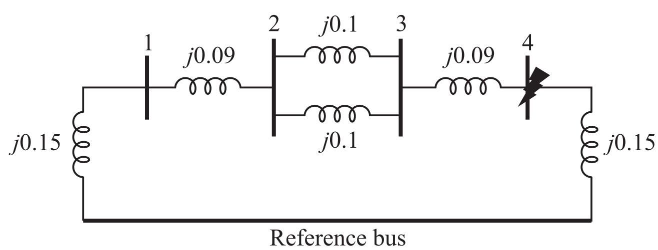






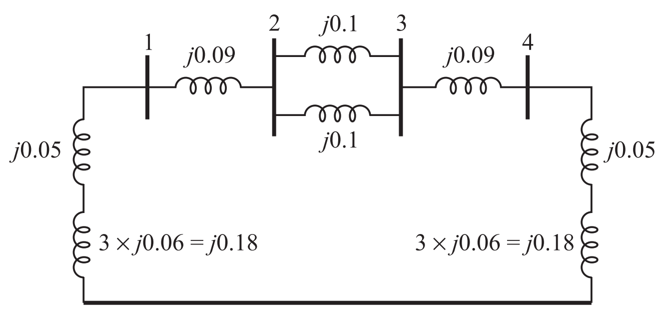
$$Z_{th} = Z_1 = j0.38 \parallel j0.15 = j1.107$$

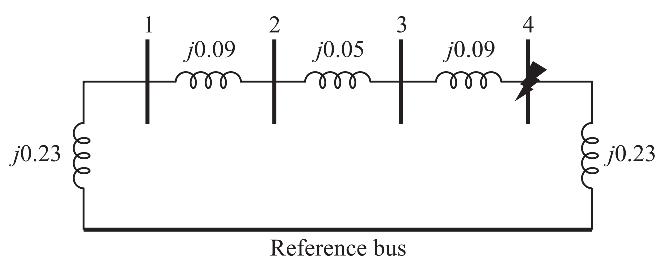
Negative sequence impedance diagram

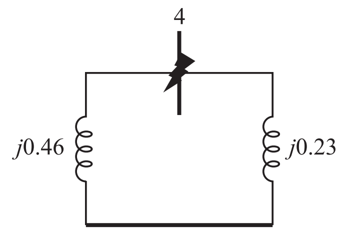


$$Z_{th} = Z_2 = j0.38 \parallel j0.15 = j0.107$$

Zero sequence impedance diagram







$$Z_{th} = Z_0 = j0.46 \,||\, j0.23 = j0.1533$$

$$E_a = 1 \text{ p.u.}$$

$$\text{Fault current } I_f = I_a = \frac{3E_a}{Z_0 + Z_1 + Z_2 + 3Z_f}$$

$$= \frac{3 \times 1.0}{j0.1533 + j0.107 + j0.107} = -j8.1677 \text{ p.u.}$$

$$\text{Base current at fault point} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_\text{b}} \times 10^3 = \frac{100}{\sqrt{3} \times 11} \times 10^3 = 5248.64 \text{ A}$$

Fault current in A = fault current in p.u. (I_f) × base current

Fault current A, $|I_f|$ = 8.1677 × 5248.64 = 42.869 kA

4.5.2 Line to Line Fault (L–L Fault)

Let us consider three-phase circuit diagram of unloaded generator fault through impedance Z_f between phases b and c as shown in Figure 4.16. The neutral of the generator is grounded through impedance. Assume that the generator is initially on no load.

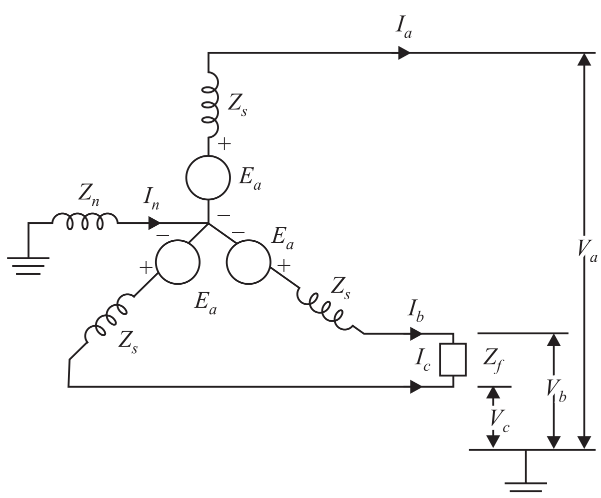


Figure 4.16 Line to line fault between b and c.

The boundary conditions at the fault point are

(i) $V_b - V_c = Z_f I_a$ (4.77)

(ii) $I_b + I_c = 0$ or $I_b = -I_c$ (4.78)

(iii) $I_a = 0$ (4.79)

The symmetrical components of currents from Eq. (4.23) can be rewritten as

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.80}$$

Substituting Eqs. (4.78) and (4.79) in Eq. (4.80), we get

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \tag{4.81}$$

From Eq. (4.81), we find that

$$I_{a0} = 0 \tag{4.82}$$

$$I_{a1} = \frac{1}{3}(a - a^2)I_b \tag{4.83}$$

$$I_{a2} = \frac{1}{3}(a^2 - a)I_b \tag{4.84}$$

Also, from Eqs. (4.83) and (4.84), we get

$$I_{a1} = -I_{a2} \tag{4.85}$$

Rewriting Eq. (4.40)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.86}$$

where zero sequence impedance, $Z_0 = Z_s + 3Z_n$
positive sequence impedance, $Z_1 = Z_s$
negative sequence impedance, $Z_2 = Z_s$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.87}$$

From Eq. (4.87)

$$V_{a0} = -Z_0 I_{a0} = -Z_0 \times 0 = 0 \tag{4.88}$$

$$V_{a1} = E_a - Z_1 I_{a1} \tag{4.89}$$

$$V_{a2} = -Z_2 I_{a2} \tag{4.90}$$

The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{4.91}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix} \tag{4.92}$$

From Eq. (4.92)

$$I_a = 0 \tag{4.93}$$

$$I_b = a^2 I_{a1} - aI_{a1} = I_{a1}(a^2 - a) \tag{4.94}$$

$$I_c = aI_{a1} - a^2 I_{a1} = I_{a1}(a - a^2) = -I_b \tag{4.95}$$

Rewriting Eq. (4.10)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{4.96}$$

Substituting Eq. (4.88) in Eq. (4.96), we get

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{4.97}$$

From Eq. (4.97)

$$V_a = V_{a1} + V_{a2} \tag{4.98}$$

$$V_b = a^2 V_{a1} + aV_{a2} \tag{4.99}$$

$$V_c = aV_{a1} + a^2 V_{a2} \tag{4.100}$$

From the boundary condition (i) $V_b - V_c = Z_f I_a$

Substituting Eqs. (4.99) and (4.100) in the above boundary conditions, we get

$$(a^2 V_{a1} + aV_{a2}) - (aV_{a1} + a^2 V_{a2}) = Z_f I_a$$

$$(a^2 - a)\,(V_{a1} - V_{a2}) = Z_f I_a \tag{4.101}$$

Substituting the Eq. (4.94) in Eq. (4.101), we get

$$(a^2 - a)\,(V_{a1} - V_{a2}) = Z_f (a^2 - a) I_{a1}$$

$$V_{a1} - V_{a2} = Z_f I_{a1} \tag{4.102}$$

Substituting Eqs. (4.89) and (4.90) in Eq. (4.102), we get

$$E_a - Z_1 I_{a1} - (-Z_2 I_{a2}) = Z_f I_{a1}$$

$$E_a = (Z_1 + Z_2 + Z_f) I_{a1}$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_f} \tag{4.103}$$

$$I_{a2} = -I_{a1} \tag{4.104}$$

$$I_{a0} = 0 \tag{4.105}$$

Therefore fault current

$$I_f = I_b = -I_c = I_{a1}(a - a^2)$$

$$I_f = I_b = -I_c = I_{a1}(-j\sqrt{3})$$

Substituting Eq. (4.103) in the above equation, we get

$$I_f = I_b = -I_c = (-j\sqrt{3})\frac{E_a}{Z_1 + Z_2 + Z_f} \tag{4.106}$$

Equations (4.85) and (4.103) can be represented by connecting the positive sequence network in parallel with negative sequence network through fault impedance as shown in the equivalent circuit of Figure 4.17. In many practical applications, Z_1 and Z_2 are the same. For bolted fault, $Z_f = 0$.

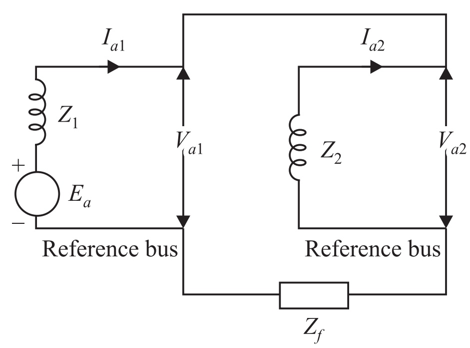


Figure 4.17 Sequence network of line to line fault.

EXAMPLE 4.9 Determine the subtransient current and the line to line voltages at the fault under subtransient conditions when a line to line fault occurs at the terminals of the generator described in Example 4.7. Assume that the generator is unloaded and operating at rated terminal voltage when the fault occurs.

Solution: $E_a = 1$ p.u.

Direct subtransient reactance, $X_d'' = Z_1 = j0.25$ p.u.

$$X_2 = Z_2 = j0.35 \text{ p.u.}$$

$$Z_f = 0$$

$$I_f = I_b = -I_c = (-j\sqrt{3})\frac{E_a}{Z_1 + Z_2 + Z_f}$$

$$\text{Fault current, } I_f = I_b = -I_c = (-j\sqrt{3})\frac{1.0}{j0.25 + j0.35} = -2.887 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_b} \times 10^3 = \frac{30}{\sqrt{3} \times 11} \times 10^3 = 1574.59 \text{ A}$$

Fault current in A = fault current in p.u. (I_f) × base current

Fault current in A, $|I_f|$ = 2.887 × 1574.59 = 4545.84 A

$$I_{a1} = -I_{a2} = \frac{E_a}{Z_1 + Z_2 + Z_f} = \frac{1.0}{j0.25 + j0.35} = -j1.667 \text{ p.u.}$$

$$I_{a0} = 0$$

$$V_{a0} = -Z_0 I_{a0} = 0 \text{ p.u.}$$

$$V_{a1} = V_{a2} = E_a - Z_1 I_{a1} = 1.0 - (j0.25)(-j1.667) = 0.584 \text{ p.u.}$$

Line to ground voltages are

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0 + 0.584 + 0.584 = 1.168 \text{ p.u.}$$

$$\begin{aligned} V_b &= V_{a0} + a^2 V_{a1} + aV_{a2} \\ &= 0 + 1\angle{-120^\circ} \times 0.584 + 1\angle 120^\circ \times (0.584) \\ &= 0 + (-0.5 - j0.866) \times 0.584 + (-0.5 + j0.866) \times (0.584) \\ &= -0.584 \text{ p.u.} \end{aligned}$$

$$\begin{aligned} V_c &= V_{a0} - aV_{a1} + a^2 V_{a2} \\ &= 0 + 1\angle 120^\circ \times 0.584 + 1\angle{-120^\circ} \times (0.584) \\ &= 0 + (-0.5 + j0.866) \times 0.584 + (-0.5 - j0.866) \times (0.584) \\ &= -0.584 \text{ p.u.} \end{aligned}$$

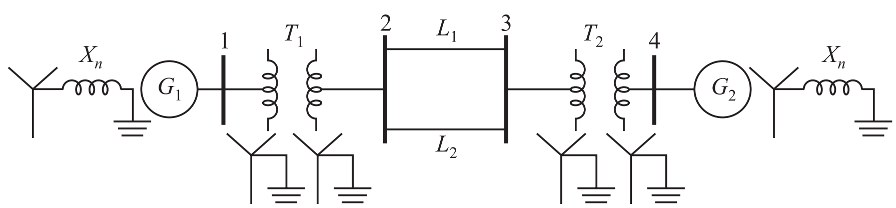
Line to line voltages are

$$\begin{aligned} V_{ab} &= V_a - V_b = 1.168 + 0.584 = 1.752\angle 0^\circ \text{ p.u.} \\ &= 1.752\angle 0^\circ \times \frac{11}{\sqrt{3}} = 11.127\angle 0^\circ \text{ kV} \end{aligned}$$

$$V_{bc} = V_b - V_c = -0.584 + 0.584 = 0 \text{ kV}$$

$$\begin{aligned} V_{ca} &= V_c - V_a = -0.584 - 1.168 = 1.752\angle 180^\circ \text{ p.u.} \\ &= 1.752\angle 180^\circ \times \frac{11}{\sqrt{3}} = 11.127\angle 180^\circ \text{ kV} \end{aligned}$$

EXAMPLE 4.10 Determine the fault current at the faulted bus for a line to line fault which occurs between phases 'b' and 'c' at bus 4 as shown in the figure.



G_1 and G_2 : 100 MVA, 20 kV, $X^+ = X^- = 15\%$; $X^0 = 4\%$ and $X_n = 6\%$
T_1 and T_2 : 100 MVA, 20/345 kV, $X_{\text{leakage}} = 9\%$
L_1 and L_2 : $X^+ = X^- = 10\%$; $X^0 = 40\%$ on a base 100 MVA.

Solution:

Base MVA, $\text{MVA}_{\text{new}} = 100$ MVA
Base kV, $\text{kV}_{\text{new}} = 345$ kV

Positive and negative reactances of transmission lines L_1 and L_2

$X_{\text{p.u.(given)}} = 0.10$ p.u., $\text{MVA}_{\text{given}} = 100$, $\text{MVA}_{\text{new}} = 100$, $\text{kV}_{\text{given}} = 345$, $\text{kV}_{\text{new}} = 345$

$$X_{\text{p.u.(new)}} = X_{\text{p.u.given}} \times \left(\frac{\text{kV}_{\text{given}}}{\text{kV}_{\text{new}}}\right)^2 \times \left(\frac{\text{MVA}_{\text{new}}}{\text{MVA}_{\text{given}}}\right)$$

$$X_{\text{p.u.(new)}} = j0.1 \times \left(\frac{345}{345}\right)^2 \times \left(\frac{100}{100}\right) = j0.1 \text{ p.u.}$$

Zero reactance of transmission lines L_1 and L_2

$X_{\text{p.u.(given)}} = 0.4$ p.u.

$$X_{\text{p.u.(new)}} = j0.4 \times \left(\frac{345}{345}\right)^2 \times \left(\frac{100}{100}\right) = j0.4 \text{ p.u.}$$

Reactance of transformers T_1 and T_2 (secondary)

$X_{\text{p.u.(given)}} = 0.09$ p.u., $\text{MVA}_{\text{given}} = 100$, $\text{MVA}_{\text{new}} = 100$, $\text{kV}_{\text{given}} = 345$
$\text{kV}_{\text{new}} = 345$

$$X_{\text{p.u.(new)}} = j0.09 \times \left(\frac{345}{345}\right)^2 \times \left(\frac{100}{100}\right) = j0.09 \text{ p.u.}$$

Base kV on LT side of transformer T_1

$$= \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$\text{Base kV on LT side of transformer } T_1 = 345 \times \frac{20}{345} = 20 \text{ kV}$$

$$\text{kV}_{\text{new}} = 20 \text{ kV}$$

Positive and negative reactances of generators G_1 and G_2

$X_{\text{p.u.(given)}} = 0.15$ p.u., $\text{MVA}_{\text{given}} = 100$, $\text{MVA}_{\text{new}} = 100$, $\text{kV}_{\text{given}} = 20$, $\text{kV}_{\text{new}} = ?$

Base kV on LT side of transformer T_1

$$= \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$\text{Base kV on LT side of transformer } T_1 = 345 \times \frac{20}{345} = 20 \text{ kV}$$

$$\text{kV}_{\text{new}} = 20 \text{ kV}$$

$$X_{\text{p.u.(new)}} = j0.15 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{100}{100}\right) = j0.15 \text{ p.u.}$$

Zero reactance of generators G_1 and G_2

$X_{\text{p.u.(given)}} = 0.04$ p.u.

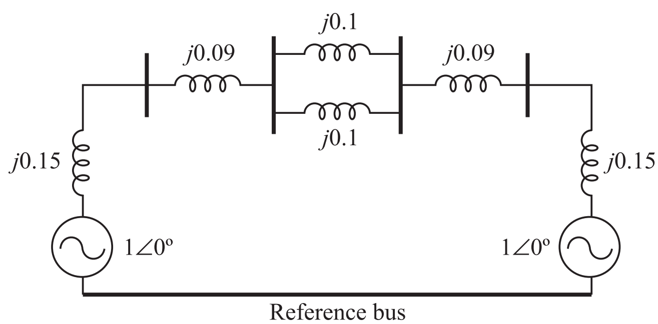
$$X_{\text{p.u.(new)}} = j0.04 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{100}{100}\right) = j0.04 \text{ p.u.}$$

Neutral reactance of generators G_1 and G_2

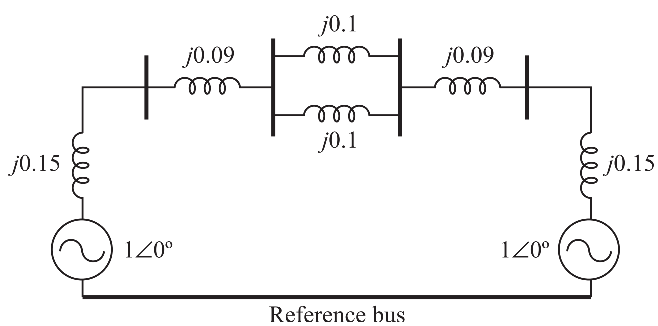
$X_{\text{p.u.(given)}} = 0.06$ p.u.

$$X_{\text{p.u.(new)}} = j0.06 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{100}{100}\right) = j0.06 \text{ p.u.}$$

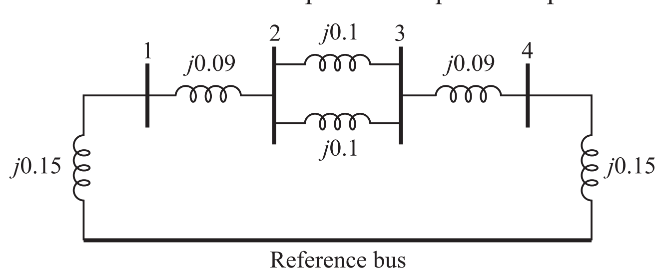
Reactance diagram

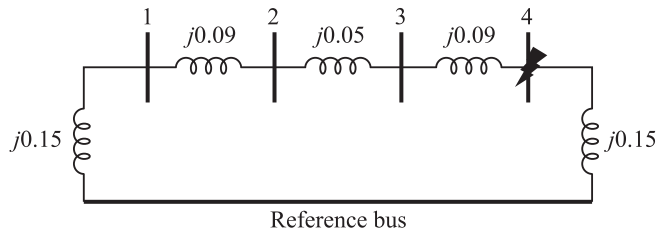


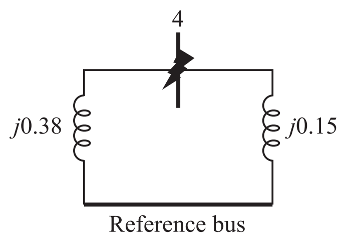
Positive sequence impedance diagram



Use Thevenin's theorem to find the positive sequence impedance Z_1.

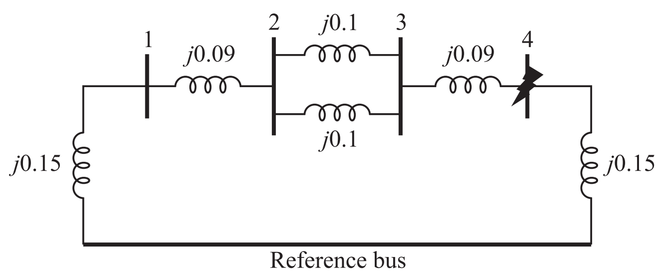






$Z_{th} = Z_1 = j0.38 \parallel j0.15 = j0.1075$

Negative sequence impedance diagram



$Z_{th} = Z_2 = j0.38 \parallel j0.15 = j0.1075$

$$E_a = 1 \text{ p.u.}$$

$$\text{Fault current, } I_f = I_b = -I_c = (-j\sqrt{3})\frac{E_a}{Z_1 + Z_2 + Z_f}$$

$$= \frac{-j\sqrt{3}(1.0)}{j0.1075 + j0.1075} = -8.056 \text{ p.u.}$$

$$\text{Base current at fault point} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_b} \times 10^3 = \frac{100}{\sqrt{3} \times 20} \times 10^3 = 2886.75 \text{ A}$$

Fault current in A = fault current in p.u. (I_f) × base current

Fault current in A, $|I_f| = 8.056 \times 2886.75 = 23.254$ kA

EXAMPLE 4.11 A 600 kVA, 11 kV, star connected three-phase alternator has positive and negative sequence reactances of 80% and 40% respectively on its own base. The neutral of the alternator is solidly earthed. When the alternator is operating on no load, with a terminal voltage of 10% in excess of the rated value, a line to line fault which is dead short circuit, occurs at the terminal. First fault currents in the alternator and also the voltage of the unfaulted phase after the occurrence of the fault.

Solution: Referring to Figure 4.16, we shall assume that the fault occurs across the phases b and c, and that the phase 'a' is the unfaulted phase; the generated voltage (on open circuit) of phase a is E_a = 1.1 p.u., taking the rated voltage as 1 p.u.

1 p.u = 11 kV (line to line)

$$\frac{11}{\sqrt{3}} \text{ kV per phase}$$

E_a = 1.1 p.u.

$$Z_1 = j0.8;\ Z_2 = j0.4$$

$$\text{Fault current } I_f = I_b = -I_c = \frac{(-j\sqrt{3})E_a}{Z_1 + Z_2} = \frac{(-j\sqrt{3}) \times 1.1}{j0.8 + j0.4} = -1.59 \text{ p.u.}$$

$$\text{Base current at fault point} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_b} \times 10^3 = \frac{600}{\sqrt{3} \times 11} \times 10^3 = 31.49 \text{ A}$$

Fault current in A = fault current in p.u. (I_f) × base current

Fault current in A, $|I_f|$ = 1.59 × 31.49 = 50 A

4.5.3 Double Line to Ground Fault (L–L–G Fault)

Figure 4.18 shows the three-phase circuit diagram of unloaded generator with a fault on phases b and c through impedance Z_f to ground. The neutral of the generator is grounded through impedance. Assume the generator is initially on no load.

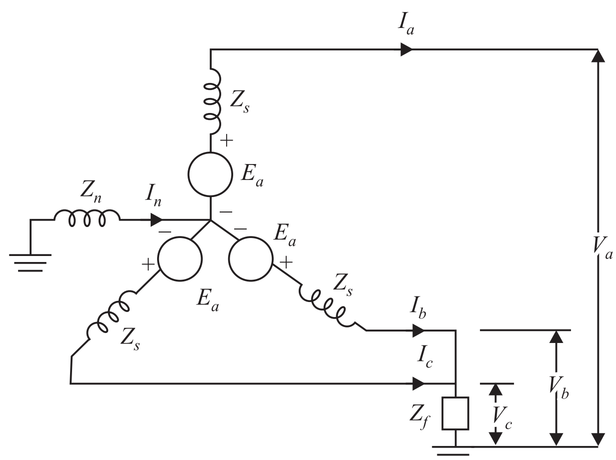


Figure 4.18 Double line to ground fault between b and c.

The boundary conditions at the fault point are

(i) $V_b = V_c = Z_f(I_b + I_c)$ (4.107)

(ii) $I_b + I_c = I_f$ (4.108)

(iii) $I_a = 0$ (4.109)

The symmetrical components of current from Eq. (4.12) can be rewritten as

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (4.110)$$

Substituting $V_b = V_c$ in Eq. (4.110)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

From the above equation we find that

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_b) = \frac{1}{3}(V_a + 2V_b)$$

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2V_b) = \frac{1}{3}(V_a + (a + a^2)V_b) = \frac{1}{3}(V_a - V_b)$$

$$V_{a2} = \frac{1}{3}(V_a + a^2V_b + aV_b) = \frac{1}{3}(V_a + (a^2 + a)V_b) = \frac{1}{3}(V_a - V_b)$$

$$V_{a1} = V_{a2} \quad (4.111)$$

The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

From the above equation

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2I_{a1} + aI_{a2}$$

$$I_c = I_{a0} + aI_{a1} + a^2I_{a2}$$

From the boundary condition

$$I_f = I_b + I_c = (I_{a0} + a^2I_{a1} + aI_{a2}) + (I_{a0} + aI_{a1} + a^2I_{a2})$$

$$= 2I_{a0} + (a^2 + a)I_{a1} + (a + a^2)I_{a2}$$

$$= 2I_{a0} + (-1)I_{a1} + (-1)I_{a2}$$

$$I_f = 2I_{a0} - (I_{a1} + I_{a2}) \quad (4.112)$$

From the boundary condition

$$I_a = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2}$$
$$0 = I_{a0} + I_{a1} + I_{a2}$$

or

$$I_{a1} + I_{a2} = -I_{a0} \tag{4.113}$$

Substituting Eq. (4.112) in Eq. (4.111), we get

$$I_f = I_b + I_c = 2I_{a0} + I_{a0} = 3I_{a0}$$

From the boundary condition

$$V_b = V_c = Z_f(I_b + I_c)$$
$$V_b = V_c = 3Z_f I_{a0} \tag{4.114}$$

Rewriting the Eq. (4.10)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$$

but

$$V_{a1} = V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + aV_{a1}$$
$$V_b = V_{a0} + (a^2 + a)V_{a1}$$
$$V_b = V_{a0} - V_{a1} \tag{4.115}$$

Equating Eqs. (4.114) and (4.115)

$$V_{a0} - V_{a1} = 3Z_f I_{a0} \tag{4.116}$$

Rewriting Eq. (4.40)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

where zero sequence impedance, $Z_0 = Z_s + 3Z_n$
positive sequence impedance, $Z_1 = Z_s$
negative sequence impedance, $Z_2 = Z_s$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

From the above equation

$$V_{a0} = -Z_0 I_{a0} \tag{4.117}$$

$$V_{a1} = E_a - Z_1 I_{a1} \tag{4.118}$$

$$V_{a2} = -Z_2 I_{a2} \tag{4.119}$$

Substituting Eqs. (4.117) and (4.118) in Eq. (4.116), we get

$$-Z_0 I_{a0} - (E_a - Z_1 I_{a1}) = 3Z_f I_{a0}$$

$$-Z_0 I_{a0} - E_a + Z_1 I_{a1} - 3Z_f I_{a0} = 0$$

$$-I_{a0}(Z_0 + 3Z_f) = E_a - Z_1 I_{a1}$$

$$I_{a0} = \frac{-E_a + Z_1 I_{a1}}{Z_0 + 3Z_f} \tag{4.120}$$

From Eq. (4.111)

$$V_{a1} = V_{a2}$$

$$E_a - Z_1 I_{a1} = -Z_2 I_{a2}$$

$$I_{a2} = \frac{-E_a + Z_1 I_{a1}}{Z_2} \tag{4.121}$$

From Eq. (4.113)

$$I_{a1} = -I_{a0} - I_{a2}$$

$$I_{a1} = -\left(\frac{-E_a + Z_1 I_{a1}}{Z_0 + 3Z_f}\right) - \left(\frac{-E_a + Z_1 I_{a1}}{Z_2}\right)$$

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}} \tag{4.122}$$

Equations (4.119) to (4.121) can be represented by connecting the positive sequence impedance in series with the parallel combination of negative sequence and zero sequence networks as shown in the equivalent circuit of Figure 4.19. The value of I_{a1} found from Eq. (4.121) is substituted in Eqs. (4.119) and (4.120), and I_{a0} and I_{a2} are found. Finally, the fault current is calculated from

$$I_f = I_b + I_c = 3I_{a0} \tag{4.123}$$

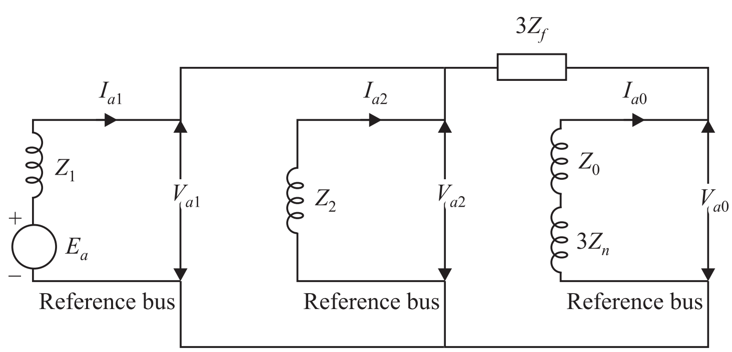


Figure 4.19 Sequence network of double line to ground fault.

EXAMPLE 4.12 Determine the subtransient current and the line to line voltages at the fault under subtransient conditions when double line to ground fault occurs at the terminals of the generator described in Example 4.7. Assume that the generator is unloaded and operating at rated terminal voltage when the fault occurs.

Solution:

$$E_a = 1 \text{ p.u.}$$

Direct subtransient reactance, $X_d'' = Z_1 = j0.25$ p.u.

$$X_2 = Z_2 = j0.35 \text{ p.u.}$$

$$X_0 = Z_0 = j0.1 \text{ p.u.}$$

$$Z_f = 0$$

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}} = \frac{1.0}{j0.25 + \dfrac{j0.35(j0.1 + 0)}{j0.35 + j0.1 + 0}} = -j3.05 \text{ p.u.}$$

$$I_{a2} = \frac{-E_a + Z_1 I_{a1}}{Z_2} = \frac{-1.0 + (j0.25)(-j3.05)}{j0.35} = j0.678 \text{ p.u.}$$

$$I_{a0} = \frac{-E_a + Z_1 I_{a1}}{Z_0 + 3Z_f} = \frac{-1.0 + (j0.25)(-j3.05)}{j0.1} = j2.375$$

$$I_f = I_b + I_c = 3I_{a0} = 3 \times j2.375 = j7.125 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\ \text{kV}_b} \times 10^3 = \frac{30}{\sqrt{3} \times 11} \times 10^3 = 1574.59 \text{ A}$$

Fault current in A = fault current in p.u. (I_f) × base current

Fault current in A, $|I_f| = 7.125 \times 1574.59 = 11.22$ kA

$$I_{a0} = I_{a0} + I_{a1} + I_{a2}$$

$$= j2.37 - j3.05 + j0.68 = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2} = -3.229 + j3.555$$

$$I_c = I_{a0} + aI_{a1} + a^2 I_{a2} = 3.229 + j3.555$$

$$V_{a0} = V_{a1} = V_{a2} = E_a - Z_1 I_{a1} = 1.0 - (j0.25)(-j3.05) = 0.237 \text{ p.u.}$$

Line to ground voltages are

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= 0.237 + 0.237 + 0.237 = 0.711 \text{ p.u.}$$

$$V_b = 0$$

$$V_c = 0$$

Line to line voltages are

$$V_{ab} = V_a - V_b = 0.711 \text{ p.u.} = 0.711 \times \frac{11}{\sqrt{3}} = 4.515\angle 0° \text{ kV}$$

$$V_{bc} = V_b - V_c = 0 \text{ kV}$$

$$V_{ca} = V_c - V_a = -0.711 \text{ p.u.} = -0.711 \times \frac{11}{\sqrt{3}} = -4.515\angle 180° \text{ kV}$$

EXAMPLE 4.13 A three-phase generator rated 11 kV, 20 MVA has a solidly grounded neutral. Its positive, negative and zero sequence reactances are 60%, 25% and 15% respectively.

(i) Determine the value of reactance that should be placed in generator neutral such that the current for single line to ground fault does not exceed the rated current.

(ii) What value of resistance in the neutral will serve the same purpose?

Solution:

(i) $E_a = 1$ p.u., $Z_1 = j0.60$ p.u., $Z_2 = j0.25$ p.u., $Z_0 = j0.15$ p.u.

$$I_f = \frac{3E_a}{Z_0 + Z_1 + Z_2 + 3Z_n}$$

$$= \frac{3 \times 1.0}{j0.15 + j0.60 + j0.25 + j3X_n} = \frac{3}{j(1.0 + 3X_n)}$$

$$= \frac{-j3}{1.0 + 3X_n} = \frac{3\angle -90°}{1.0 + 3X_n}$$

where X_n is the reactance connected to the neutral. Since the rated current is 1.0 p.u., therefore, ground fault current is also 1.0 p.u.

$$|I_f| = 1.0 = \frac{3}{1.0 + 3X_n}$$

$$X_n = 0.666 \text{ p.u.}$$

$$X_n \text{ (in Ohm)} = X_n \text{ p.u.} \times \frac{\text{kV}_b^2}{\text{MVA}_b}$$

$$= 0.66 \times \frac{11^2}{20} = 4\ \Omega$$

(ii) To find R_n

$$I_f = \frac{3E_a}{Z_0 + Z_1 + Z_2 + 3R_n}$$

$$= \frac{3 \times 1.0}{j0.15 + j0.60 + j0.25 + 3R_n}$$

$$= \frac{3}{3R_n + j1.0}$$

where R_n is the reactance connected to the neutral. Since the rated current is 1.0 p.u., therefore, ground fault current is also 1.0 p.u.

$$|I_f| = 1.0 = \frac{3}{j1.0 + 3R_n}$$

$$1.0 = \frac{3}{\sqrt{1.0 + 9R_n^2}}$$

$$R_n = 0.9428 \text{ p.u.}$$

$$R_n \text{ (in Ohm)} = R_n \text{ p.u.} \times \frac{\text{kV}_b^2}{\text{MVA}_b}$$

$$= 0.9428 \times \frac{11^2}{20} = 5.7\ \Omega$$

$$R_n = 5.7\ \Omega$$

EXAMPLE 4.14 An alternator of negligible resistance, with solidly grounded neutral having rated voltage at no load condition is subjected to different types of fault at its terminal. The p.u. values of the magnitude of the fault currents are (i) three-phase fault = 4.0 p.u. (ii) line to ground fault = 4.2857 p.u. (iii) line to line fault = 2.8868 p.u. Determine the p.u. values of the sequence reactances of the machine.

Solution:

(i) Three-phase fault, I_f = 4.0 p.u.

$$E_a = E'_g = E''_g = 1.0 \text{ p.u.}$$

$$|I_f| = \frac{|E_a|}{X'_d}$$

$$X'_d = \frac{|E_a|}{|I_f|} = \frac{1.0}{4.0} = 0.25 \text{ p.u.}$$

$$Z_1 = X'_d = 0.25 \text{ p.u.}$$

(ii) Line to line fault, I_f = 2.8868 p.u.

$$I_f = (-j\sqrt{3})\frac{E_a}{Z_1 + Z_2}$$

$$I_f = (-j\sqrt{3})\frac{1.0}{j0.25 + jZ_2}$$

$$|I_f| = \frac{\sqrt{3}}{0.25 + Z_2}$$

$$2.8868 = \frac{\sqrt{3}}{0.25 + Z_2}$$

$$0.25 + Z_2 = \frac{\sqrt{3}}{2.8868}$$

$$0.25 + Z_2 = 0.5999$$

$$Z_2 = 0.5999 - 0.25 = 0.35$$
$$Z_2 = 0.35 \text{ p.u.}$$

(iii) Single line to ground fault, $I_f = 4.2857$ p.u.

$$I_f = \frac{3E_a}{Z_0 + Z_1 + Z_2}$$
$$Z_0 + Z_1 + Z_2 = \frac{|3E_a|}{|I_f|}$$
$$Z_0 + 0.25 + 0.35 = \frac{3}{4.2857}$$
$$Z_0 = 0.7 - 0.25 - 0.35 = 0.1$$
$$Z_0 = 0.1 \text{ p.u.}$$

Review Questions

Part-A

1. What are the symmetrical components of a three-phase system?
2. What are the positive sequence components?
3. What are the negative sequence components?
4. What are the zero sequence components?
5. What is the sequence operator?
6. Write down the equations to convert the symmetrical components into the unbalanced phase currents. (or) How to determine the unbalanced currents from the symmetrical currents?
7. Write down the equations to convert the unbalanced phase currents into symmetrical components. (or) How to determine the symmetrical currents from the unbalanced currents?
8. What are the sequence impedance and sequence network?

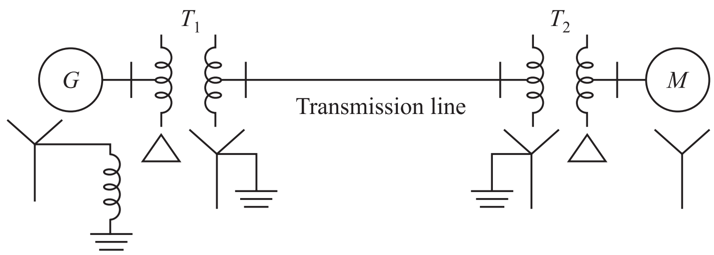
Part-B

1. The phase voltages across a certain load are given as follows

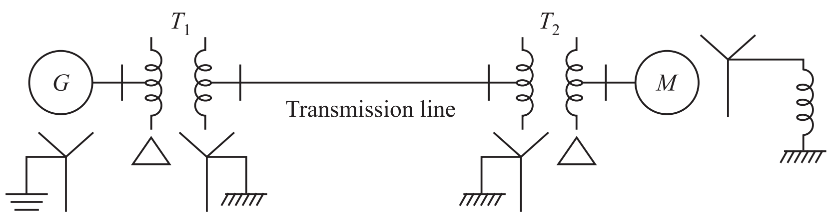
$$V_a = 176 - j132 \text{ V}$$
$$V_b = -128 - j96 \text{ V}$$
$$V_c = -160 + j100 \text{ V}$$

   Compute the positive, negative and zero sequence components of voltage.
2. A balanced delta connected load is connected to a three-phase system and a current of 15 A is supplied to it. If the fuse of one of the lines melts, compute the symmetrical components of line currents.

3. Draw the zero sequence network of the power system as shown in the figure below.

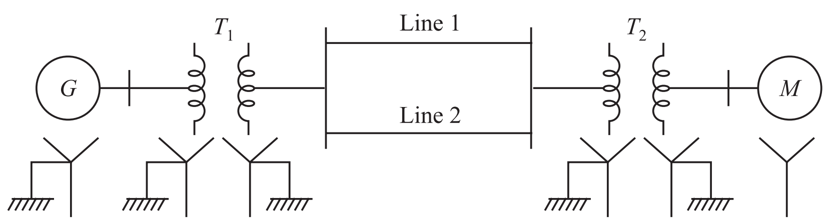


4. Draw the zero sequence network of the power system as shown in the figure below.

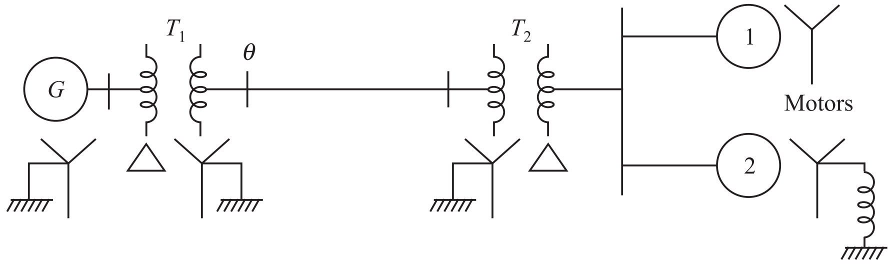


5. Draw the zero sequence network of the power system as shown in the figure below. Data are given below.

G: $X_{g0} = 0.05$ p.u.	M: $X_{m0} = 0.03$ p.u.
T_1: $X_{T1} = 0.12$ p.u.	T_2: $X_{T2} = 0.10$ p.u.
Line 1: $X_{L10} = 0.70$ p.u.	Line 2: $X_{L20} = 0.70$ p.u.



6. A 50 MVA, 11 kV, synchronous generator has a subtransient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in the figure below. The motors have rated inputs of 30 MVA and 15 MVA, both 10 kV, with 25% subtransient reactance. The three-phase transformers are both rated 60 MVA, 10.8/121 kV, with leakage reactance of 10% each. Assume zero sequence reactance for the generator and motors of 6% each. The current limiting reactors of 2.5 Ω each are connected in the neutral of the generator and the motor number is 2. The zero sequence reactance of the transmission line is 300 Ω. The series reactance of the line is 100 Ω. Draw the positive, negative and zero sequence networks.



7. A 30 MVA, 13.2 kV synchronous generator has a solidly grounded neutral. Its positive, negative and zero sequence impedances are 0.30, 0.40 and 0.05 p.u. respectively.
   (a) What value of reactance must be placed in the generator neutral so that the fault current for a line to ground fault of zero fault impedance shall not exceed the rated line current?
   (b) What value of resistance in the neutral will serve the same purpose?
   (c) What value of reactance must be placed in the neutral of the generator to restrict the fault current to ground to rated line current for a double line to ground fault?
   (d) What will be the magnitude of the line current when the ground current is restricted as above?
   (e) As the reactance in the neutral is indefinitely increased, what are the limiting values of the line current?

8. Two alternators are operating in parallel and supplying a synchronous motor which is receiving 60 MW power at 0.8 power factor lagging at 6.0 kV. The single line diagram for this system is given in the figure below. Data are given below. Compute the fault current when a single line to ground fault occurs at the middle of the line through a fault resistance of 4.033 Ω. Data are given below:
   G_1 and G_2: 100 MVA, 11 kV, $X^+ = 0.2$ p.u., $X^- = X^\circ = 0.1$ p.u.
   T_1: 180 MVA, 11.5/115 kV, $X = 0.1$ p.u.
   T_2: 170 MVA, 6.6/115 kV, $X = 0.1$ p.u.
   M = 160 MVA, 6.3 kV, $X^+ = X^- = 0.3$ p.u., $X^\circ = 0.1$ p.u.
   Line: $X^+ = X^- = 30.25\ \Omega$, $X^\circ = 60.5\ \Omega$